

Analysis of Thermoelectric Generator Using Solar Energy

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ABSTRACT

A complete theoretical analysis is applied to thermoelectric generator. Finite difference method is applied to governing differential employed in analysis. In addition to this, solar thermal calculations are also done in order to understand feasibility of employing solar thermal with the system.

Keywords: DC electricity, n-type semiconductor thermoelectric generator

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LITERATURE SURVEY

The performance of thermoelectric generators (TEGs) is affected by the internal irreversibility's such as joule's heat and heat leak. TEGs are composed simply of p-type and n-type semiconductors between high-temperature T(1) and low-temperature T(2) source, and convert part of heat absorbed to DC electricity. The externally and internally irreversible heat engine model is used to analyze the performance of a TEG. The performance of a TEG is sensitively dependent on the degree of external irreversibility's [1]. Many have attempted to take the heat which is released as exhaust heat from vehicles and use it as a source (input) to TEG. It may be possible to recover significant amount of exhaust waste with a thermoelectric device. TEG performance is better on the exhaust pipe than on radiator because heat is more at exhaust pipe. The output power and efficiency increase significantly by changing the convection heat transfer

coefficient of the high-temperature side than that of low-temperature side. Maximum output power and efficiency of a TEG are obtained when external resistance is greater than internal resistance [2].

Also, computer tools are used to accurately simulate the thermal and electrical dynamics of a real thermal electric power generation system. This paper presents an innovative and powerful computer tool to simulate real thermoelectric power generation system. A program is designed by using SIMULINK MATLAB, and experimental results are compared [3].

An analytical model of the solar heat pipe thermoelectric generator (SHP-TEG) system has been made for the condition of constant solar irradiation. Based on the analytical model, different simulations have been carried out to know the performance and design of the SHP-TEG.

To get the maximum power condition of constant solar irradiation, the matched external load resistance is a little larger than the TEG internal electrical resistance [4]. Various methods were used in which fabrication of a TEG by bulk semiconductor, by organic semiconductors or by flexible TE chips, but these methods were limited due to their weight problems or large surface areas.

So, we have developed a method which solves the surface area problem and also reduces thermal contact influence. Therefore, heat transfer analysis is been used, where a flexible thermoelectric generator based on thin-film thermoelectric junctions on flexible fibers is used.

Alternate surfaces are heated to check the thermoelectric properties of the fiber which results in the formation of temperature gradient. Resistivity measured experimentally is near to the theoretical values.

The temperature gradient of a TEG is almost constant but can be varied by the segment length. Comparison of the silica fibers with the polymers is also done and was found beneficial. Voltage and power output are measured as a function of temperature difference between the junctions [5]. Some of them have used finite-time thermodynamics along with non-equilibrium thermodynamics assuming Newton's laws. Variations over power output and thermal efficiency with respect to electric current are also carried out. The equations derived can also be used for quoting heat transfer surface area of the heat exchanger. Also, ratio of thermoelectric elements for maximum efficiency and power is also conducted.

By varying the number of thermoelectric elements on the top and bottom, a different power output is obtained which can be seen

from the characteristics graph of power vs. current, which is mostly a parabola. From the derivations and graphs, we can say that, by varying upper thermoelectric elements, the power output and efficiency decrease and vice versa. While by varying lower thermoelectric elements, it increases and vice versa. Thus, one should keep in mind while designing practical thermoelectric generator that it is economical [6].

In this paper, the study of performance of parallel thermoelectric generator and its output characteristics are the main objectives. A model whose theoretical analysis and calculation are based on thermodynamics principles, semiconductor theory and law of conservation of energy is used. Many models comprise TEG in series, so here parallel network of TEG is considered. It is also assumed that all the parameters of TEG are constant. As TEG works on Seebeck, Peltier and Thomson effects, it converts thermal energy and produces electrical energy, but in a small amount, i.e. the output power is low. So, a parallel combination is used to increase its output performance. To measure the temperatures, a thermocouple is inserted in the cooling plates. The thermal resistance and the influence of contact effect play an important role.

To reduce the influence of contact effect, the thermal contact resistance of the TE module of heat source and the heat sink are reduced so as to increase the output performance of TEG. Here the theoretical value is more than the experimental value, which is because of the heat condition of thermocouple. Hence the output performances can be predicted [7].

The electrical parameters of TEG under different temperature changes are also dealt. Here we have developed a method to determine the parameters under load conditions by making the internal

temperature difference as a function of load current. The construction is very simple which includes several p- and n-type plates squeezed between ceramic plates and all are connected in series with copper layer.

Effective thermal resistance is the series of the parasitic resistance formed by ceramic plates, copper resistance and the effective thermal resistance formed by parallel combination from all p- and n-type elements.

Thermoelectric generator acts as a Thévenin voltage source for the model and is directly proportional to the Seebeck coefficient with a temperature difference between hot and cold junctions and the Thévenin resistance of the pellets. In this paper, the method that we proposed to determine the parameters is by certain equations and their verification by their experimental *IV* curves. The values of Seebeck coefficient, the internal temperature difference and other electrical parameters are well fitted in comparison with their experimental results.

We also see that the temperature difference decreases linearly with load current which is as a result of Peltier effect. It causes temperature drop and this is well derived in this TE model [8].

A engine which is reversible internally but irreversible externally and which exchanges heat with its surroundings through a finite temperature difference is known as endo reversible heat engine. A TEG is used for producing electric power directly from waste heat in small amount.

Here we have discussed the thermoelectric generator whose real working process is compared by reversible process. The Seebeck effect is brought into the picture. Two semiconductor devices, p- and n-types, are connected to the hot and cold

junctions through metal contact. Temperatures at heat source and heat sink are T_h and T_l , whereas hot and cold junctions are T_w and T_c .

As seen, two legs of the generator are p- and n-type, so heat flows from them and current flows in series with them. A generator is designed in such a way to maximize the output power. Hence this paper deals with real waste heat thermoelectric generator considering both internal and external irreversibility effects.

Estimation of more specific power and efficiency than the ideal TEG can be made by this method [9]. To supply power to remote microsystems that have no physical connections is also done. Power output is proportional to the cube of the vibration frequency, so device isn't suited for small frequency and also proportional to mass deflection.

This device generates electricity from mechanical energy when embedded in vibrating medium. Power output of $1 \mu\text{W}$ and 0.1 mW at 70 and 330 Hz, respectively, is predicted for a device, assuming deflection of $50 \mu\text{m}$. Its main limitation is that its size which limits maximum travel distance of mass. Mass deflection can be controlled by designing spring which matches the vibration frequency and making damping factors small. It is suitable for microsystems with power requirements that depend on frequencies [10].

For waste heat recovery in industry area, a low-temperature waste heat TEG setup has been constructed. It has found that thermoelectric module in series, expanding heat sink surface area in a proper range and enhancing cold-side heat transfer capacity in a proper range can be employed.

The heat transfer irreversibility affects thermoelectric generator performance [11].

It is proposed to calculate the optional parameters of the bottoming cycle when objectives are function of net output power and volumetric expansion. It is also suggested that TEG-ORC system recovers waste heat from engines, because TEG can extend the temperature range of heat source and thereby improve the security and fuel economy of engines.

This system has many benefits compared with only ORC system in receiving waste heat of engine exhaust gases. The effect of relative flow direction, TEG scale, the highest temperature, condensation temperature, evaporator pressure and efficiency system performance are investigated and optimized. Under subcritical and supercritical condition, evaporator, turbine, IHE, condenser, pump and TEG for the system are investigated. Energetic and exergetic calculations are conducted.

Based on system size, economy and heat increase on overall efficiency analyses. Parametric and exergetic performance investigation are done as follows:

- i. This system is most effective at $P_{\text{evp}} = 5.5 \text{ MPa}$ and $T_{\text{cond}} = 303 \text{ K}$. Net power output 27.01 kW maximum efficiency gets higher for 41% (without WHR system) to 45.7%. In this also, TEG is also important. Here R123 is used in the evaporation.
- ii. The thermodynamic irreversibility in evaporator, turbine, IHE, condenser, pump and TEG for system indicates highest efficiency about 46.03% when efficiency of IHE is 0.8. The energy loss in the TEG is the largest. High evaporator pressure and IHE with high efficiency help gain higher system performance [12].

To accurately and efficiently analyze thermoelectric device, a new set of ANSYS coupled-field elements is enabled by users. The finite element is formulation, in

addition to Joule heating, includes Seebeck, Peltier and Thomson effects [13]. The humans are developing faster, so fossil fuels are limited, and hence new and different sources are there like solar energy, hydropower energy, and thermoelectrical energy. In this, thermoelectrical energy is more favorable. It has wider application and high performance, and micro-TEG is fabricated by material deposited using MEMS. The structure is investigated by finite-element methods with 3-D model. The unit composed of P–N pair; in thermoelectrical unit, thermoelements are in parallel and series electrically. If we use P–N-type materials like Sb_2Te_3 and Bi_2Te_3 , they have the finest property at room temperature. The widely used materials in MEMS are silicon (Si) and copper (Cu) which used as substrates and electrodes. If we are investigating the effect of substrate, the thickness is higher and thermal loss is more. Heat flux is also decreased. So, low efficiency is more. For designing the micro-TEG, the parameters which are used for design is optimized by analyzing 3-D model [14]. The output of the power and expression efficiency for semiconductors generator (thermoelectric) are made of multi-element which can be derived with some consideration of heat transfer between the generator and reservoirs.

It shows that heat transfer irreversibility does not affect the performance of TEG; this effect is useful for the analysis of real generator with composition of multi-element and number of thermoelectric elements. It affects the performance of generator, so the selection of thermoelectric element is done in favor of optimization between efficiency and the output of power. So the best performance is obtained by optimization of generator carried out in the 2. Further, optimization one is internal and another is external. These elements are used for optimizing couple leg, and further, the use for heat transfer and optimizing

external load and internal resistance material [15].

This technology is in solid-state energy conversion way which transfers thermal energy into electrical energy. This power has no moving parts. It has wider applications. The applications are wide but they are limited because of law of conversion efficiency. In industrial work, the waste heat is there; it is not recycled. The TEG has a good advantage of low-grade waste heat recovery. To study the flow rate, properties of fluid temperature and power, effect of heat transfer between heat reservoir and thermoelectric device is analyzed. It is a multi-element when resistance is equal to inner resistance, and then the maximum power is obtained. The efficiency of the TEG does not change, but increases with increase in temperature. If we have to promote the performance of the system, increase in waste heat temperature is must, If we see the irreversibility process between its module and reservoirs. It improves OR refines the system performance [16].

METHODOLOGY

$$\rho \frac{D}{Dt} \left(e + \frac{1}{2} u_i u_i \right) = \bar{v}q + \rho g_1 u_1 + \frac{\partial y}{\partial x} (u_i \tau_{ji}) \quad (1)$$

$$\rho \frac{DE}{Dt} = -(vq) + \tau_{ji} \frac{\partial u_i}{\partial x_j} \quad (2)$$

$$\pi_{jidi} = 2\mu_{dij} d_{ij} - \frac{2}{3} \mu \overline{(v \cdot u)^2} \quad (3)$$

$$\rho \frac{De}{Dt} = -(\nabla \bar{q}) - p \nabla u + \phi \quad (4)$$

$$\rho Dh/Dt = -(\nabla q) + \frac{dp}{dt} + \phi \quad (5)$$

Using Equations (1)–(5), the final form of thermal energy balance equations will be shown. Tds equations will be used to derive entropy balance equations. Maxwell's relationships will also be used.

Also, C_v form of thermal energy balance equation will be converted to C_p form of thermal energy balance equation. On the left-hand side, we are having energies in storage, and in right-hand side, energy in transit, in which the first term contains energy due to heat transfer and the second term contains energy due to work done, and the third term is work done due to surface forces. And after subtracting mechanical energy balance equation, we get the final form of thermal energy balance equation.

Here E is the internal energy, and when the equation is recasted, it can be written in the form of enthalpy, and so the equation contains H for enthalpy.

$$Tds = dh - vdp$$

Here enthalpy is on the right-hand side, and on the left-hand side, we are having entropy by dividing by dt with respect to time.

Multiplying both sides by density which is the reciprocal of specific volume, and by equating the value of Dh/Dt in Equation (5),

$$Tds = dh - vdp \rightarrow T$$

$$Tds = dh - vdp \rightarrow T \frac{ds}{dt} = \frac{dh}{dt} - v \frac{dp}{dt}$$

$$\frac{dh}{dt} = t \frac{ds}{dt} + v \frac{dp}{dt} \rightarrow \rho \frac{Dh}{Dt} = \rho \left\{ T \frac{Ds}{Dt} + V \frac{DP}{Dt} \right\}$$

Hence in the new form of energy, the left-hand side is the energy in the form of storage, while the right-hand side terms contain energy in the form of transit with one term due to heat transfer and the other as viscous dissipation. Hence it is the entropy form of thermal energy balance equation.

From the enthalpy form of balance of thermal energy equation,

$$\rho \frac{Dh}{Dt} = -(\nabla \bar{q}) + \frac{DP}{Dt} + \phi$$

$$\rho T \frac{Ds}{Dt} = -\nabla \bar{q} + \phi$$

This is the entropy form of thermal balance equation.

Now we will be trying to derive the Cv form of conversation of thermal energy balance equation. Internal energy is the function of volume and temperature. So it can be written in the form as follows:

$$e = e(T, v)$$

$$de = \left(\frac{\partial e}{\partial T}\right) v dt + \left(\frac{\partial e}{\partial v}\right) t dv = cvdT + \left(\frac{\partial e}{\partial v}\right) t dv$$

Here the first term determines the amount if energy required to change the temperature per unit degree at constant volume and it is equivalent to write it as Cv. Again by using the thermodynamic relationship of Tds equation, by little modification by diving by dv and using Maxwell's relationships and putting it in the final equation,

$$\left(\frac{\partial e}{\partial v}\right) t = T \left(\frac{\partial s}{\partial v}\right) t - p$$

$$\left(\frac{\partial s}{\partial v}\right) t = \left(\frac{\partial P}{\partial T}\right) v$$

$$De = \left(\frac{\partial e}{\partial T}\right) v dt + \left(\frac{\partial e}{\partial v}\right) t dv = cvdT + \left(\frac{\partial e}{\partial v}\right) t dv$$

$$\left(\frac{\partial e}{\partial v}\right) t = \left(\frac{\partial s}{\partial v}\right) t - P$$

$$\left(\frac{\partial e}{\partial v}\right) T = t \left(\frac{\partial p}{\partial T}\right) v - P$$

$$De = CvdT + \left[T \left(\frac{\partial P}{\partial T}\right) - P \right] dv$$

$$CvdT + \left[P - T \left(\frac{\partial p}{\partial T}\right) v \right] d\rho/\rho^2$$

$$\frac{de}{dT} = cv \frac{dT}{dt} = \left[P - T \left(\frac{\partial P}{\partial T}\right) v \right] 1/\rho^2 \frac{d\rho}{dt}$$

Now by replacing v by 1\r specific density, the final equation will be as follows. Hence the right-hand side contains the energy due to transit in which the first term is due to heat transfer, the second is hydrostatics and the third is the deviatoric part. Hence by further simplification, the final equation is shown as below. This equation can be

called as the Cv form of conversion of thermal energy balance equation.

Now proceeding further in converting the above equation into Cp form, which is nothing but heat at constant pressure. As we know, the enthalpy is the function of temperature and pressure. Now replacing the change in temperature per unit degree at constant pressure (in the enthalpy equation) by Cp, similarly as we had done in the Cv derivation, but here we will divide by dh. Using Maxwell's relationship, the equation can be modified as follows:

$$h = h(T, P)$$

$$dh = \frac{\partial h}{\partial T} pdt + \frac{\partial h}{\partial P} t dp = Cpdt + \frac{\partial h}{\partial P} t dp$$

$$Tds = dh - vdp$$

$$T \frac{ds}{dp} = \frac{dh}{dp} - v$$

$$\frac{\partial h}{\partial P} t = T \frac{\partial s}{\partial P} t + \bar{v}$$

Using Maxwell's relationship,

$$\left(\frac{\partial s}{\partial P}\right) t = - \left(\frac{\partial v}{\partial T}\right) p$$

$$\frac{dh}{dp} t = -T \left(\frac{\partial v}{\partial T}\right) p + v$$

To simplify it, we had considered beta which is equal to change in volume per unit change in temperature at constant pressure:

$$\left(\frac{\partial h}{\partial P}\right) t = -T \left(\frac{\partial v}{\partial T}\right) p + v = \left[v - \frac{1}{v} \frac{\partial v}{\partial T} T + 1 \right]$$

$$= v(1 - \beta T)$$

$$\beta = \frac{1}{v} \frac{\partial v}{\partial T} T$$

$$dh = cpdT + (1 - \beta T) \frac{dp}{\rho}$$

Further, by simplification and calculation and using Fourier's laws, the final equation is shown below and this equation is called the Cp form of balance of thermal energy:

$$\frac{\partial h}{\partial P} t = T \frac{\partial s}{\partial P} t + v$$

$$\left(\frac{\partial s}{\partial P}\right) t = - \left(\frac{\partial v}{\partial T}\right) p$$

$$\left(\frac{\partial h}{\partial p}\right)_t = -T \left(\frac{\partial v}{\partial T}\right)_p + v \left[v - \frac{1}{v} \frac{\partial v}{\partial t} T + 1\right]$$

$$=v(1 - \beta T)$$

$$\frac{dh}{dt} = cp \frac{dT}{dt} + (1 - \beta T/\rho) \frac{dP}{dT}$$

$$\rho \frac{dh}{dt} = \rho cp \frac{dT}{dt} (1 - \beta T) \frac{dP}{dT}$$

$$\rho cp \frac{dT}{dt} + (1 - \beta T) \frac{dP}{dT} = \frac{dP}{dT} - \nabla \bar{q} + \bar{\phi}$$

$$\rho cp \frac{dT}{dt} = \beta T \frac{dP}{dT} + \nabla \cdot (k \nabla T) + \bar{\phi}$$

This is the C_p form of balance of thermal energy. The following analysis is applied to the three domains of TEG. There are three separate domains which include the following:

	$\sigma E - \sigma \alpha \nabla T + \tau \frac{\partial E}{\partial t} = J_o$	
(I) $\rho C_v \frac{\partial T}{\partial t} = \nabla(k \nabla T)$	(II) $\rho C_v \frac{\partial T}{\partial t} = \nabla[(k + \sigma \alpha^2 T) \nabla T - \sigma \alpha T E] + \sigma E^2 - \sigma \alpha E \nabla T$	(III) $\rho C_p \frac{\partial T}{\partial t} = \nabla(k \nabla T)$
A	B	C D

I and III represent heat source and sink, and II represents thermoelectric material. We know that

$$J = \sigma E - \sigma \alpha \nabla T \frac{A}{m^2}$$

$$q = \pi J - k \nabla T \frac{W}{m^2}$$

where E and T are the electric field and temperature, respectively, α is the Seebeck coefficient, π is the Peltier coefficient, σ is the electrical conductivity and k is the thermal conductivity.

Any material having both, electrical and heat conduction,

1. Temperature gradient causes an electric field to develop in the absence of electrical current, and
2. An electric field causes a thermal gradient to develop in the absence of thermal current.

In finite-volume method, application to the above governing differential equations, the properties of material and electrical conductivity are assumed to be constant. This allows considerably simple application of equations in finite-volume approach.

In finite-volume approach of computational fluid dynamics, each domain is divided into several small elements of size Δx whose center is X_i and end points are $X_{i-1/2}$ and $X_{i+1/2}$. The temperature in space at particular instant of time is approximated as the average temperature in domain of an element which is shown as follows:

$$\begin{aligned} \bar{T}_i(t) &= \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} T(x, t) dx \\ \bar{T}_i(t) &= \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} T(x, t) dx \approx \frac{T(x_{i+1/2}, t) - T(x_{i-1/2}, t)}{\Delta x} \approx \frac{\bar{T}_{i+1}(t) - \bar{T}_{i-1}(t)}{2\Delta x} \\ \bar{T}_i(t) &= \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial^2 T}{\partial x^2}(x, t) dx \approx \frac{\frac{\partial T}{\partial x}(x_{i+1/2}, t) - \frac{\partial T}{\partial x}(x_{i-1/2}, t)}{\Delta x} \\ &\approx \frac{\frac{\bar{T}_{i+1}(t) - \bar{T}_i(t)}{\Delta x} - \frac{\bar{T}_i(t) - \bar{T}_{i-1}(t)}{\Delta x}}{\Delta x} \\ &= \frac{\bar{T}_{i+1}(t) - 2\bar{T}_i(t) - \bar{T}_{i-1}(t)}{(\Delta x)^2} \end{aligned}$$

In order to discretize equations in domains 1 and 3 which has the heat equations

$$\begin{aligned} \rho C_v \frac{\partial T}{\partial t} &= \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) \\ \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \rho C_v \frac{\partial T}{\partial t} dx &= \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx \\ \rho C_v \frac{\partial \bar{T}_i}{\partial t} &= \frac{k}{\Delta x} \left[\frac{\partial T}{\partial x} \Big|_{x_{i+1/2}} - \frac{\partial T}{\partial x} \Big|_{x_{i-1/2}} \right] \\ \rho C_v \frac{\partial \bar{T}_i}{\partial t} &= \frac{k}{\Delta x} \left[\left(\frac{\bar{T}_{i+1} - \bar{T}_i}{\Delta x} \right) - \left(\frac{\bar{T}_i - \bar{T}_{i-1}}{\Delta x} \right) \right] \end{aligned}$$

The following equations show how equation in the second section is discretized:

$$\begin{aligned} \rho C_v \frac{\partial T}{\partial t} &= \frac{\partial}{\partial x} [(k + \sigma \alpha^2 T) \frac{\partial T}{\partial x} - \sigma \alpha T E] + \sigma E^2 - \sigma \alpha E \frac{\partial T}{\partial x} \\ \in \frac{\partial E}{\partial x} &= J_0 - \sigma E + \sigma \alpha \frac{\partial T}{\partial x} \\ \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \rho C_v \frac{\partial T}{\partial t} dx &= \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \sigma E^2 - \sigma \alpha E \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} [(k + \sigma \alpha^2 T) \frac{\partial T}{\partial x} - \sigma \alpha T E] dx \\ \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \in \frac{\partial E}{\partial x} dx &= \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} J_0 - \sigma E + \sigma \alpha \frac{\partial T}{\partial x} dx \end{aligned}$$

$$\begin{aligned} \rho C_v \frac{\partial \bar{T}_i}{\partial t} &= \sigma \bar{E}_i^2 - \sigma \bar{\alpha}_i \bar{E}_i \frac{T_{i+\frac{1}{2}} - T_{i-\frac{1}{2}}}{\Delta x} + \frac{k}{\Delta x} \left[\frac{\partial T}{\partial x_{i+\frac{1}{2}}} - \frac{\partial T}{\partial x_{i-\frac{1}{2}}} \right] + \frac{\sigma \bar{\alpha}_i^2}{\Delta x} \left[T \frac{\partial T}{\partial x_{i+\frac{1}{2}}} - T \frac{\partial T}{\partial x_{i-\frac{1}{2}}} \right] \\ &\quad - \frac{\sigma \bar{\alpha}_i}{\Delta x} \left[(TE)_{i+\frac{1}{2}} - (TE)_{i-\frac{1}{2}} \right] \\ \in \frac{\partial \bar{E}_i}{\partial x} &= J_0 - \sigma \bar{E}_i + \sigma \bar{\alpha}_i \frac{T_{i+\frac{1}{2}} - T_{i-\frac{1}{2}}}{\Delta x} \\ \rho C_v \frac{\partial \bar{T}_i}{\partial t} &= \sigma \bar{E}_i^2 - \sigma \bar{\alpha}_i \bar{E}_i \frac{\bar{T}_{i+1} - \bar{T}_{i-1}}{2\Delta x} + \frac{k}{\Delta x} \left[\frac{\bar{T}_{i+1} - 2\bar{T}_i + \bar{T}_{i-1}}{\Delta x} \right] \\ &\quad + \frac{\sigma \bar{\alpha}_i^2}{\Delta x} \left[\frac{\bar{T}_{i+1} + \bar{T}_i}{2} \frac{\bar{T}_{i+1} - \bar{T}_i}{\Delta x} - \frac{\bar{T}_i + \bar{T}_{i-1}}{2} \frac{\bar{T}_{i-1} - \bar{T}_i}{\Delta x} \right] \\ &\quad - \frac{\sigma \bar{\alpha}_i}{\Delta x} \left[\frac{\bar{T}_{i+1} + \bar{T}_i}{2} \frac{\bar{E}_{i+1} + \bar{E}_i}{2} - \frac{\bar{T}_i + \bar{T}_{i-1}}{2} \frac{\bar{E}_i + \bar{E}_{i-1}}{2} \right] \\ \in \frac{\partial \bar{E}_i}{\partial t} &= J_0 - \sigma \bar{E}_i + \sigma \bar{\alpha}_i \frac{T_{i+1} - T_{i-1}}{2\Delta x} \end{aligned}$$

This completes the discretization of governing differential equations (applied to domain 2 out of domain 3), and next comes the boundary condition application in domains 1 and 3, which is applied as follows:

$$\begin{aligned} \rho C_v \frac{\partial T}{\partial t} &= \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) \\ \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \rho C_v \frac{\partial T}{\partial t} dx &= \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx \\ \rho C_v \frac{\partial \bar{T}_i}{\partial t} &= \frac{k}{\Delta x} \left[\frac{\partial T}{\partial x_{i+1/2}} - \frac{\partial T}{\partial x_{i-1/2}} \right] \end{aligned}$$

On the left-hand side boundary, heat is applied and so heat flux or Neumann boundary condition is applied to the left-hand boundary condition.

The heat flux equation from Fourier's law of heat conduction is given as follows:

$$-k \frac{\partial T}{\partial x} = q_0$$

This is applied to heat equation:

$$\rho C_v \frac{\partial \bar{T}_i}{\partial t} = \frac{k}{\Delta x} \left[\left(\frac{\bar{T}_{i+1} - \bar{T}_i}{\Delta x} \right) - q_0 \right]$$

The above completes boundary condition application on the left-hand side of domain 1.

The next is the application of right-hand side boundary condition:

Since the right-hand side is exposed to atmosphere (since we want temperature gradient in order to generate the electricity which is realm of TEG), we apply direct boundary condition as follows:

$$\rho C_v \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right)$$

$$\frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \rho C_v \frac{\partial T}{\partial t} dx = \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx$$

$$\rho C_v \frac{\partial \bar{T}_i}{\partial t} = \frac{k}{\Delta x} \left[\frac{\partial T}{\partial x_{i+1/2}} - \frac{\partial T}{\partial x_{i-1/2}} \right]$$

$T = T_{amb}$, so $\frac{\partial T}{\partial x_{i+1/2}}$

$$\rho C_v \frac{\partial \bar{T}_i}{\partial t} = \frac{k}{\Delta x} \left[\frac{T_{amb} - \bar{T}_{i-1}}{\Delta x/2} - \frac{\bar{T}_i - \bar{T}_{i-1}}{\Delta x} \right]$$

(I) $\rho C_v \frac{\partial T}{\partial t} = \nabla(k\nabla T)$	(II) $\sigma E - \sigma \alpha \nabla T + \tau \frac{\partial E}{\partial t} = J_0$ $\rho C_v \frac{\partial T}{\partial t} = \nabla[(k + \sigma \alpha^2 T)\nabla T - \sigma \alpha T E] + \sigma E^2 - \sigma \alpha E \nabla T$	(III) $\rho C_v \frac{\partial T}{\partial t} = \nabla(k\nabla T)$
A	B	C

The above description completes A and D boundary condition applications, and the following description gives explanation of B and C boundary condition applications, which is done as follows:

$$\rho C_v \frac{\partial \bar{T}_i}{\partial t} = \sigma \bar{E}_i^2 - \sigma \bar{\alpha}_i \bar{E}_i \frac{\bar{T}_{i+1} - \bar{T}_{i-1}}{2\Delta x} + \frac{k}{\Delta x} \left[\frac{\bar{T}_{i+1} - 2\bar{T}_i + \bar{T}_{i-1}}{\Delta x} \right]$$

$$+ \frac{\sigma \bar{\alpha}_i^2}{\Delta x} \left[\frac{\bar{T}_{i+1} + \bar{T}_i}{2} \frac{\bar{T}_{i+1} - \bar{T}_i}{\Delta x} - \frac{\bar{T}_i + \bar{T}_{i-1}}{2} \frac{\bar{T}_{i-1} - \bar{T}_i}{\Delta x} \right]$$

$$- \frac{\sigma \bar{\alpha}_i}{\Delta x} \left[\frac{\bar{T}_{i+1} + \bar{T}_i}{2} \frac{\bar{E}_{i+1} + \bar{E}_i}{2} - \frac{\bar{T}_i + \bar{T}_{i-1}}{2} \frac{\bar{E}_i + \bar{E}_{i-1}}{2} \right]$$

$$\in \frac{\partial \bar{E}_i}{\partial t} = J_0 - \sigma \bar{E}_i + \sigma \bar{\alpha}_i \frac{\bar{T}_{i+1} - \bar{T}_{i-1}}{2\Delta x}$$

Electric field may be written in terms of voltage as follows:

$$W(x, t) = E \Delta x$$

W is the difference in voltage across an element which has a distance Δx ,

$$\rho C_v \frac{\partial \bar{T}_i}{\partial t} = \frac{\sigma \bar{W}_i^2}{\Delta x^2} - \sigma \bar{\alpha}_i \frac{\bar{W}_i}{\Delta x} \frac{\bar{T}_{i+1} - \bar{T}_{i-1}}{2\Delta x} + \frac{k}{\Delta x} \left[\frac{\bar{T}_{i+1} - 2\bar{T}_i + \bar{T}_{i-1}}{\Delta x} \right]$$

$$+ \frac{\sigma \bar{\alpha}_i^2}{\Delta x} \left[\frac{\bar{T}_{i+1} + \bar{T}_i}{2} \frac{\bar{T}_{i+1} - \bar{T}_i}{\Delta x} - \frac{\bar{T}_i + \bar{T}_{i-1}}{2} \frac{\bar{T}_{i-1} - \bar{T}_i}{\Delta x} \right]$$

$$- \frac{\sigma \bar{\alpha}_i}{\Delta x} \left[\frac{\bar{T}_{i+1} + \bar{T}_i}{2} \frac{\bar{W}_{i+1} + \bar{W}_i}{2\Delta x} - \frac{\bar{T}_i + \bar{T}_{i-1}}{2} \frac{\bar{W}_i + \bar{W}_{i-1}}{2\Delta x} \right]$$

$$\frac{\varepsilon}{x\Delta} \frac{\partial \bar{W}_i}{\partial t} = J_0 - \sigma \frac{\bar{W}_i}{\Delta x} + \sigma \bar{\alpha}_i \frac{T_{i+1} - T_{i-1}}{2\Delta x}$$

Multiplying temperature equation by $A \Delta x$ and voltage equation by A (where A is a parameter required for calculations),

$$\begin{aligned} C_{th} \frac{\partial \bar{T}_i}{\partial t} = & \frac{\bar{W}_i^2}{\Delta R_e} - \bar{\alpha}_i \frac{\bar{W}_i}{\Delta R_e} \frac{\bar{T}_{i+1} - \bar{T}_{i-1}}{2} + \frac{1}{\Delta R_{th}} (\bar{T}_{i+1} - 2\bar{T}_i + \bar{T}_{i-1}) \\ & + \frac{\bar{\alpha}_i^2}{2\Delta R_e} (\bar{T}_{i+1}^2 - 2\bar{T}_i^2 + \bar{T}_{i-1}^2) \\ & - \frac{\bar{\alpha}_i}{4\Delta R_e} [(\bar{T}_{i+1} + \bar{T}_i)(\bar{W}_{i+1} + \bar{W}_i) - (\bar{T}_i + \bar{T}_{i-1})(\bar{W}_i + \bar{W}_{i-1})] \end{aligned}$$

$$C_e \frac{\partial \bar{W}_i}{\partial t} = I_0 - \frac{\bar{W}_i}{\Delta R_e} + \bar{\alpha}_i \frac{T_{i+1} - T_{i-1}}{2\Delta R_e}$$

$$C_{th} = A\Delta x\rho C_v$$

$$\Delta R_{th} = \frac{\Delta x}{kA}$$

$$\Delta R_e = \frac{\Delta x}{\sigma A}$$

$$C_e = \frac{A \varepsilon}{\Delta x}$$

$$I_0 = AJ_0$$

These equations are solved in MATLAB.

RESULTS AND DISCUSSION

The following results were obtained from calculations performed by MATLAB software as shown in Table 1.

Table 1 shows the variation of voltage with various parameters of TEG as well as external parameters like external resistance, ambient temperature, etc. The voltage values are the steady-state values with respect to changes in parameters.

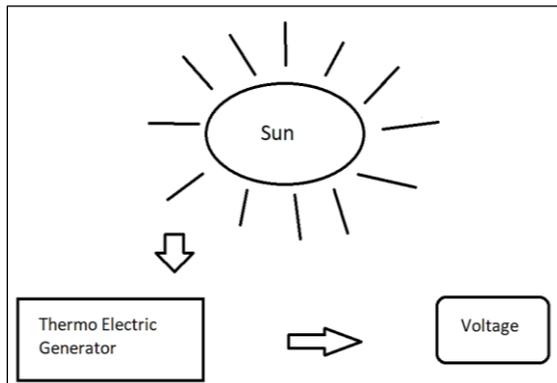
It shows that if proper selection of parameters is taken considering the effects of external parameters, then the required voltage can be obtained which is necessary for various applications.

Table 1. Calculations performed by MATLAB software.

Current (A)	Voltage variation (V)	Resistance (ohm)	Voltage variation (V)	Ambient temperature (K)	Voltage variation	Heat (J)	Voltage variation (V)
0.05	2.209869	3.5	2.054606	280	1.974932	50	2.445079
0.1	1.96701	4	2.00644	290	1.973942	60	2.941972
0.15	1.724785	4.5	1.957896	300	1.972903	70	3.4302
0.2	1.483189	5	1.909068	310	1.971066	80	3.918479

Table 2. Calculation on solar thermal.

Length (m)	Width (m)	Cross area (m ²)	Heat gained from collector (kW)
3	2	6	3.75
3.4	2.4	8.16	4.43
3.9	2.9	11.31	6.55
4.3	3.3	14.19	7.88
4.7	3.9	18.33	9.746
5	4	20	10.56
5.4	4.2	22.68	12.66
5.9	4.9	28.91	17.34
6.7	5.3	35.51	23.26

**Fig. 1.** System employed.

The results in Table 2 were obtained by performing calculation on solar thermal. Table 2 shows heat available from solar collector to be given to generators of system. If we assume that solar thermal can contribute 23 kW of heat supplied to generators of system, then it can give full heat requirement to the system for generators. This can save full running costs of heater yearly which may be significant if components are maintained regularly. Figure 1 shows the system employed.

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