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Effect of Consideration in Euler Beam Theory

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ABSTRACT

The purpose of this paper is to improve the results of Euler beam theory. Euler beam theory can be used to determine the deflection of beam. But in some cases cross-section of beam is not flat-ended, and Euler beam theory only consider the vertical loading, so the effect of curvature is occurred due to normal loading. The method to determine the curvature effect has been developed in this paper, and verified with the software simulation.

Keywords: Euler beam theory, non-flat-ended, the curvature effect, software simulation

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INTRODUCTION

Euler beam theory can be used to determine the deflection of any kind of beam, and one of my research work has been done on Euler beam theory to determine the deflection of tapered cantilever beam [1]. Erasmo et al [2] and Ferdinand et al. [3] have described formula to determine are moment of inertia about any section, and basic theories related beam. Many of researchers have utilized Euler Bernoulli concept for beam application. Malukhin et al. [4] have applied the Euler beam theory to determine the deflection of collet by considering each jaw of collet as a cantilever beam. Farid et al. [5] applied the Euler Beam concept for calculating the fundamental natural frequency for non-uniform cross section or tapered cantilever beam. Frieman and Kosmatka [6] have also applied Euler and FEA based approach for non-uniform section beam. Al-Gahtani and Khan [7] have used generalized Euler Bernoulli equation for calculating the deflection of non-prismatic beam having parabolic variation with transverse uniform loading condition, and this method has been applied for determining the deflection of bridge having transverse loading and parabolic section. Shooshtari and Khajavi [8] and Reza [9] have calculated the stiffness matrix and shape function for non-prismatic beam by Euler Bernoulli and Timoshenko formulation. So in previous researches, many of the researchers have worked on Euler Bernoulli theory used for determining deflection and stiffness of cantilever beam with prismatic and non-prismatic section. None of researcher have worked on curvature effect of Euler beam theory, so its scope for research to develop such theory.

Curvature Effect

As shown in Figure 1 the case of cross section of beam is flat ended, loading is only occurred in vertical direction and Euler beam theory will also consider the loading as vertical direction, while in case of non-flat ended cross section shown in Figure 2, loading occurs in normal direction, so there are two components like vertical and horizontal components, but Euler theory considers only vertical loading, while in actual case the force N will share the horizontal component also. So, actual vertical loading will be less.

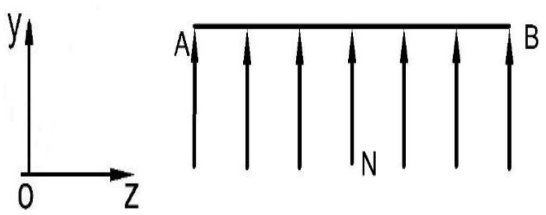

Fig. 1. *Flat ended cross-section.*

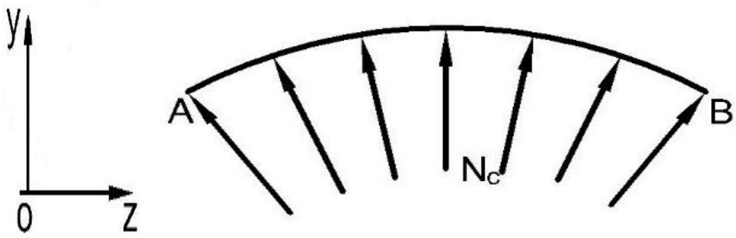

Fig. 2. *Non-flat ended cross-section.*

If we compare the case 2 for Euler beam theory then it can be see that in Euler theory, whole part of N is converted into vertical force while in actual scenario, the component of N will be horizontal and vertical, so the actual vertical force would be lesser than Euler assumption, and vertical deflection will be lesser than Euler beam assumption.

Euler beam theory will consider flat shape instead of curvature shape, so the vertical actual load will be lesser than the load assumed by Euler beam theory. Because curvature loading creates some amount of horizontal load.

Analytical Method to Determine the Curvature Effect

Figure 3 shows the non-flat ended cantilever beam, normal and uniform loading N is applied at lower surface of beam. Where L = length of beam, a = width of beam. And cross-section area of cantilever beam (non-flat ended) is shown in Figure 4.

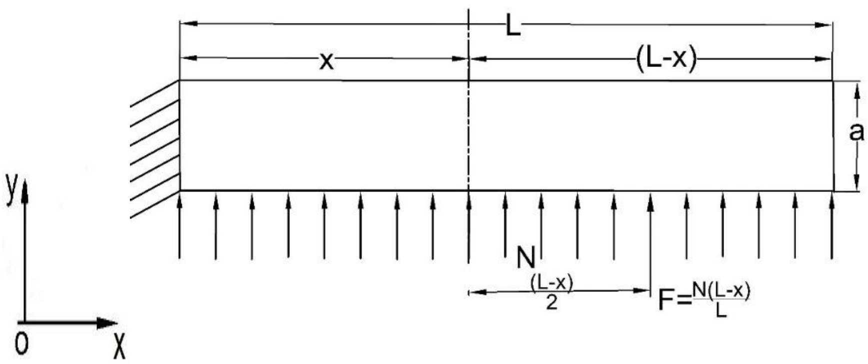

Fig. 3. *Cantilever beam (non-flat ended).*

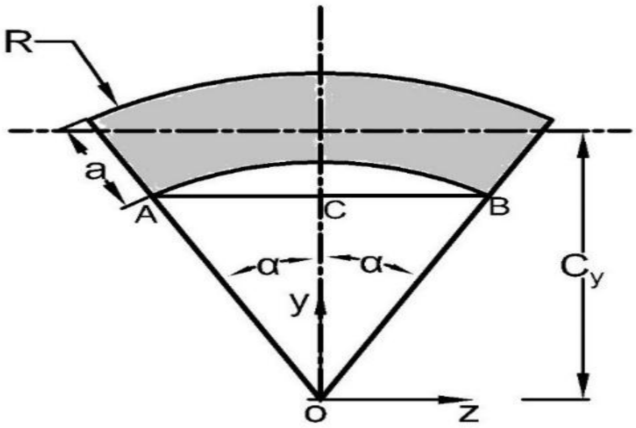

Fig. 4. *Cross-section area of cantilever beam (non-flat ended).*

Using Euler equation deflection of cantilever beam can be calculated as follows [2]:

$$\frac{d^2y}{dx^2} = \frac{M_x}{I_x E} \qquad (1)$$

where y(x) = deflection function, M_x = Bending moment about C.G. axis at any distance x due to force F_x, I_x = Area moment of inertia of any section, E = Modulus of Elasticity for isotropic material.

Force acting on section at distance x is

$$F_x = \frac{N\,(L-x)}{L} \qquad (2)$$

Perpendicular distance from normal force F_x for any section at distance x from the Figure 3 is,

$$d_x = \frac{N\,(L-x)}{L}. \qquad (3)$$

M_x = *Force x Distance*

$$M_x = \frac{N(L-x)}{L}\, x\, \frac{L-x}{2}$$

$$M_x = \frac{N(L-x)^2}{2L}. \quad (4)$$

Area Moment of inertia for section I_x, (calculated using basic concepts of mechanics [2, 3])

$$I_{z(C.G.)} = \frac{(R^4-(R-a)^4)\left[\theta+\frac{\sin(2\theta)}{2}\right]}{4} - \frac{4\sin^2(\theta)\left(R^3-(R-a)^3\right)^2}{9\theta(R^2-(R-a)^2)} \quad (5)$$

where, a = width of beam, R = Radius of curvature, θ = half arc angle and C_y = Center distance from z axes.

Solution of the equation (1) by putting value of M_x and I_z

$y(x) = \frac{N}{2LI_zE}\left[\frac{(L-x)^4}{12} + \frac{xL^3}{3} - \frac{L^4}{12}\right]$ (6)

where N = **l**oading without considering curvature effect, N_c = effective vertical load by considering curvature effect, A = area under flat end, A_c = actual area under loading.

If both the method giving equal deflection in vertical direction then vertical load should be equal and for that in both the cases pressures should be equal,

$P = P_c$

So,

$\frac{N}{A} = \frac{N_c}{A_c}$

Using ΔOCB,

$$sin\,(\alpha) = \frac{BC}{OB}$$

$BC = OB\ x\ \ sin\,(\alpha)$

$OB = R - a$

So,

$BC = (\mathrm{R} - \mathrm{a})\ x\ \ sin\,(\alpha)$

$AB = 2\ x\ BC$

$AB = 2(R - a)\ x\ \ sin\,(\alpha)$

$A = AB\ x\ L$

$A = 2(\mathrm{R} - \mathrm{a})\ sin\,(\alpha)\ \ L$

$A_c = Arc\ length\ AB\ x\ L$

$A_c = (\mathrm{R} - \mathrm{a})\ x\ 2\alpha\ x\ L$

$A_c = 2(\mathrm{R} - \mathrm{a})\ \alpha\ L$

$$\frac{N}{2(\mathrm{R} - \mathrm{a})\ sin\,(\alpha)\ x\ L} = \frac{N_c}{2\ (\mathrm{R} - \mathrm{a})\ \alpha\ L}$$

Effective load,

$$N_c = \frac{2N(\mathrm{R} - \mathrm{a})\ sin\,(\alpha)\ x\ L}{2\ (\mathrm{R} - \mathrm{a})\ \alpha\ L}$$

$N_c = \frac{N\ sin\,(\alpha)}{\alpha}$ (7)

$N_c = C_e\ N$ and $y_c\,(x) = C_e\ y(x)$

where y_c (x) = actual deflection produce by N load, y(x) = deflection obtained by Euler method, C_e = curvature constant.

$\boldsymbol{C_e} = \frac{\boldsymbol{sin\,(\alpha)}}{\boldsymbol{\alpha}}$ (8)

Case Study

Let us consider the cantilever beam of similar section having loading of 1000 N, R=400 mm, a=100 mm, E=200 GPa and α = 22.5 degree.

Solution y_{max} using Euler equation,

y_{max} = 0.026729751 mm

Solution by considering curvature effect,

$$\alpha = 22.5x\frac{PI}{180}$$

Pi = 3.141593

α =0.392699 radian

$C_e = 0.009744954$

$$N_c = C_e N$$

$$N_c = \frac{1000 \sin(0.392699)}{0.392699}$$

$N_c = 974.4954\ N$

$Y_{max\ (Actual)}$ = 0.026048019 mm

Difference,

$= Y_{max}\ (Actual) - Y_{max}\ (Using\ Euler)$

$= 0.026048019 - 0.026729751$

$= -0.000681732\ mm$

Difference in %

$$= \frac{Ymax\ (Actual) - Ymax\ (Using\ Euler)}{Ymax\ (Actual)} x100$$

$$= \frac{0.026048019 - 0.026729751}{0.000681732} x100$$

$= 2.61721246 44\%$

VERIFICATION USING CREO-SIMULATE SOFTWARE

Creo-simulate is advanced software to perform FEA analysis on mechanical structure. The model of cantilever beam has been prepared using Creo-parametric by taking similar parameters of case study (L = 1000mm, a = 100mm, θ = 22.5 degree, E = 200GPa), and analysis has been performed in Creo-Simulate software. Here analysis has been performed by considering two cases: (1) without considering the effect of curvature and (2) considering the effect of curvature.

Case 1: Vertical loading (1000 N)

As shown in Figure 5. Loading is considered as a vertical only. The whole part of 1000 N normal force has been utilized for creating vertical loading. Here, results are obtained near to the Euler beam approach without considering curvature effect.

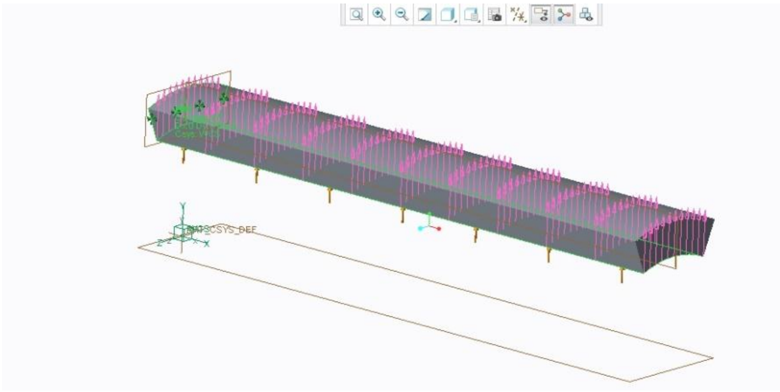

Fig. 5. *Vertical loading.*

Figure 6 shows the results plotting of Creo-simulate. The maximum deflection is occurred at free end of the beam, and minimum deflection is occurred near to the fixed end.

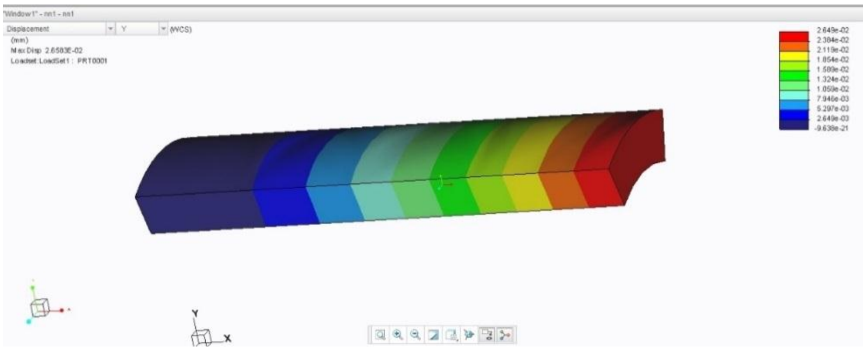

Fig.6. *Deflection using Creo-simulate (without curvature).*

Y_{max} = 0.026583mm

As shown in Figure 7, loading is considered as a normal to surface. The whole part of 1000 N normal force has been utilized for creating vertical loading

Case 2: Normal loading (1000 N) and horizontal loading. Here, results are obtained near to the Euler beam approach with considering the curvature effect.

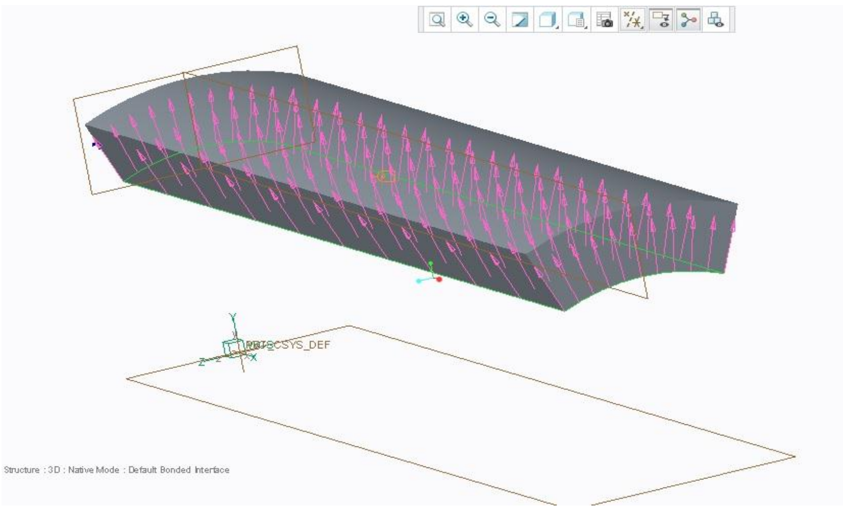

Fig. 7. *Normal loading.*

Figure 8 shows the results plotting of Creo-simulate. The maximum deflection is occurred at free end of the beam, and minimum deflection is occurred near to the fixed end.

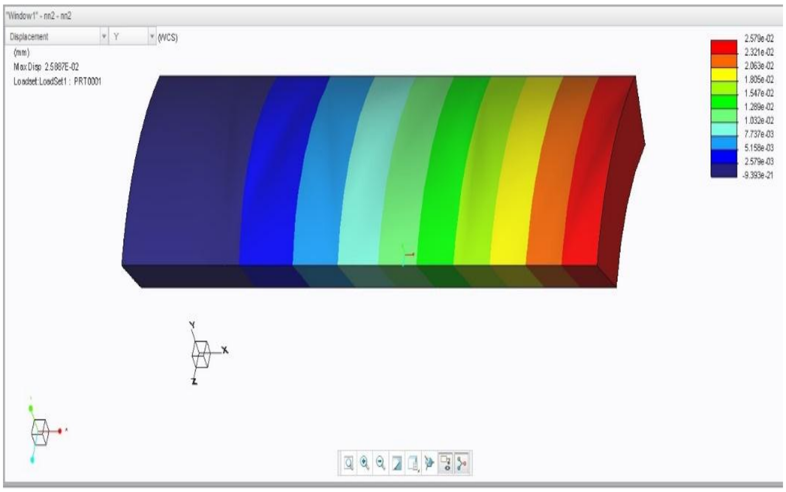

Fig. 8. *Deflection using Creo-simulate (with curvature).*

Y_{max} = 0.025887mm

Difference

$= Y_{max\ (actual)} - Y_{max\ (Euler)}$

= 0.025887 - 0.026583

=–0.000696mm

Difference in %

$$= \frac{\text{Ymax (Actual)} - \text{Ymax (Euler)}}{\text{Ymax (Actual)}} x100$$

$$= \frac{0.025887 - 0.026583}{0.025887} x100$$

= 2.6886081712%

COMPARISON OF ANALYTICAL METHOD AND CREO-SIMULATE RESULTS

The following comparison is done to validate the scope of analytical method. Analytical method and Creo-simulate has been compared based on same dimensions and loading conditions for isotropic material.

Similarity, Table 1 shows that analytical method have closed aggrement toward the Creo-Simulate results, so analytical method has been successfully verified.

Table 1. *Comparison of various methods of deflection.*

No	Method	Y_{max} (mm)	Y_{max} mm (with curvature effect)	Difference (mm)	Difference percentage
1	Analytical approach	0.026729	0.026048	0.000682	2.62%
2	Creo-simulate	0.026583	0.025887	0.000696	2.68%
	Total difference	**0.000146**	**0.000161**	**0.000014**	**0.06%**

CONCLUSION

Euler beam theory can be used to determine the deflection of any type of cantilever beam. But the results obtained by Euler beam theory doesn't consider the effect of curvature, that's why for the non-flat ended beam, slight variation is come in deflection. In this paper, analytical method for curvature effect has been successfully developed, and verified using Creo-simulate. This method improves the accuracy of Euler beam for non-flat ended beam.

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